Q1. The reflection spectrum (Figure 1) of a periodic quarter-wave dielectric stack structure was measured using an optical spectrum analyzer (OSA) at normal incidence. Peak reflectivity of 90% occurs at 1550 nm, and the first reflection null on the long-wavelength side of the reflection band occurs at 1560 nm. Find:

(1) The optical length of the periodic structure (i.e. $\bar{n}L$, $\bar{n}$ being the average refractive index, $L$ being the physical length of the structure.) [3 marks]

(2) The relative refractive index contrast $\Delta n/\bar{n}$, $\Delta n$ being the index difference. [3 marks]

(3) The number of periods. [3 marks]

(4) Sketch in Figure 1 the reflection spectrum if the number of periods is doubled (hence the total length is also doubled), with all other parameters unchanged [4 marks]

(Note 1: You may make reasonable approximations, such as those made in the coupled-mode theory.)

(Note 2: Your sketch should be qualitatively correct in terms of $\lambda_0$, $R_{\text{max}}$, bandwidth, and null positions, when compared to the original spectrum.)

![Reflection Spectrum](https://via.placeholder.com/150)
Q2. An asymmetric semiconductor slab waveguide structure is shown in Figure 2. The indices of the materials are $n_1=3.49$, $n_2=3.51$, and $n_3=3.50$. Consider $1.55\mu m$ light of TE polarization.

(1) What is the minimum thickness of the guiding layer in order to support at least one mode? [2 marks]

Assume the guide thickness is $4.7\mu m$.

(2) How many guided modes can propagate in this structure? [2 marks]

(3) Sketch qualitatively the field and intensity distributions for all guided modes. (Please sketch them separately and label clearly. Exaggerate asymmetry in your sketch.) [2 marks]

(4) Write the E field expression for the lowest-order TE mode, using the coordinate system specified in Figure 2. [3 marks]

(5) What is the speed of energy transport for the lowest-order TE mode? (Neglect material dispersion, i.e., treat $n_1$, $n_2$ and $n_3$ as wavelength independent). [4 marks]

Figure 2. The asymmetric slab waveguide structure
Answer either Q3 or Q4, NOT both.

Q3. The wave equation governing the TE modes in a planar dielectric waveguide is given by:

\[
\left( \frac{\partial^2}{\partial x^2} + \left( \frac{\omega}{c} n(x) \right)^2 - \beta^2 \right) E_y(x) = 0
\]

Derive from Maxwell’s equations the wave equation for the TM modes. [6 marks]
Answer either Q3 or Q4, NOT both.

Q4. Consider the structure illustrated in Figure 3, with $n_1 > n_2$. Assume the dielectrics are lossless, and the incident angle $\theta$ is greater than the critical angle,

1. What is the maximum transmission achievable from A to B, if you have full control of the parameters $a$ and $b$? [2 marks]

2. Find the condition(s) under which maximum transmission occurs. [4 marks]